ACCURATELY PREDICTING TRANSIENT FLUID FORCES IN PIPING SYSTEMS

Part 1: Fundamentals

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Scott A. Lang P.E.
Applied Flow Technology
Colorado Springs, Colorado, USA

Trey W. Walters P.E.
Applied Flow Technology
Colorado Springs, Colorado, USA

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ACCURATELY PREDICTING TRANSIENT FLUID FORCES IN PIPING SYSTEMS PART 1: FUNDAMENTALS

Scott A. Lang, PE
Applied Flow Technology
Colorado Springs, CO

Trey W. Walters, PE
Applied Flow Technology
Colorado Springs, CO

ABSTRACT

Changes in the operation of piping systems – like valve closures or pump starts – propagate pressure waves that travel at acoustic velocity throughout the fluid. These pressure waves have considerable effect on forces, potentially generating dynamic loads upwards of 10,000 lbf (50 kN) in common configurations.

Some estimation methods used in industry for estimating transient forces neglect terms that may be important in some cases. Calculating forces due to these transients without simplification for transient liquid or gas flow is presented here in detail.

Keywords: unsteady, forces, piping stress, dynamic load

NOMENCLATURE

F	Generic Force
R	Reaction Force
a	Wavespeed
F_{est}	Estimated Maximum Force
C_L	Characteristic Ratio
$\mathbb{R}$	Flow Resistance

1. INTRODUCTION

1.1 Motivation

Many advanced techniques exist for determining stress and strain on piping systems. Safe use of these tools is only possible if the driving forces are determined accurately.

Force calculations rapidly become complex even in static systems and fluid systems are rarely static. Pressure is often considered the dominant driver of force and thus the only effect of interest, an overly simplified approach which neglects real physical effects.

Fluid transients make the situation even more complex. Sudden changes in system operation – like valve closures or pump starts – propagate pressure waves throughout the entire system which directly impact the load that must be carried by piping. Pressure waves are strongly coupled to velocity waves, which means momentum and frictional losses are also transient. These must all be considered for a full representation of transient force.

The complexity of calculating forces due to fluid transients has led to, at worst, the effect being neglected entirely or being handled by simple correction factors. Even those aware of the transient nature often consider only pressure terms. In some cases, such simplifications are reasonable, but in other cases the simple approach considerably miscalculates the actual forces.

Beyond the desire of the design engineer to avoid excessive stresses, limits for occasional loads are specified in standards such as ASME B31. ASME B31.1-101.5.1 specifically indicates that loads due to fluid transients *shall* be considered [1]. Similar statements are made in ASME B31.3 and B31.4 [2] [3]. To ensure the code is met, a method with high confidence should be used.

The authors are of the opinion that, as with most fluid transient analyses, engineers should not speculate on whether such simplifications are acceptable for their system and should instead remove all doubt by computing the true force with the method outlined herein.

1.2 Historical Background

The fundamentals that will be discussed here are not new. In fact, the approach taken is entirely based on simple principles such as Newton's Laws. Applying these laws correctly to an assembly of piping under transient conditions has, however, not been adequately described in a plain manner to the knowledge of the authors.

Discussion of component reactions due to specific effects, such as the reaction due to flow around a bend, or shear due to wall friction, can be found in nearly any introductory fluids textbook [4]. These cases are often considered in academic isolation, making application in the field challenging. Great care must be taken to maintain a consistent system of forces and account for all terms of interest – something introductory texts do not cover.

A true accounting of the transient force must consider all transient terms, not just component forces that exist at the control volume boundaries. Across an acoustic, mass flows are not equal, and the temporal effect must be considered. Accounting for these transient behaviors is a complicated step often omitted from a typical analysis but is not a new concept; the method was noted in some detail in [5]. Unfortunately, the theory found in [5] was used primarily to develop a simple estimation, which can introduce significant error [6].

One of the most common approaches to calculating transient forces is to first determine accurate transient flow results and then simply use transient pressures at the control volume boundaries multiplied by local flow areas to determine boundary component forces. The overall reaction force is calculated as the sum of these boundary component forces. While well-intentioned, using endpoint pressures only is effectively a steady-state approach taken at every step because it neglects terms such as the acceleration of fluid within the control volume or behavior of the fluid through devices.

1.3 Assumptions

The approach outlined here is not without assumptions of its own. Most importantly, deformation of piping is not considered – the entire assembly is assumed to be completely rigid.

Results can therefore be regarded as the reaction forces required to keep a piping assembly in place. Specific reactions at supports can be determined for statically determinant systems. The presented method serves only to determine the overall reaction – effects of determinacy or deformation are out of scope.

External elements such as heavy valve stems, pump skids, or wind are not considered. Forces due to thermal expansion, multi-phase flows, or other complex fluid dynamics are also neglected.

Flow and all related parameters within the piping are considered in a one-dimensional context. That is, pressure, velocity, or any other parameter, are considered averaged values throughout the cross-section of flow.

1.4 Numerical Examples

The examples discussed in Part 1 are intentionally simple in nature and serve only to enhance discussion on the theory and method behind the force calculation. For more in depth examples and results, see Part 2 [7].

1.5 Reactions vs. Forces on a Piping Assembly

This paper is focused on the determination of *reaction* forces on a piping assembly. These are forces required to keep a system from moving, which must be provided by external supports – for example, a reaction due to a mass on a surface is the normal force acting *upward* to counteract weight.

Piping stress analysts may be concerned with the net force acting *on* a piping assembly – which is equal in magnitude but opposite in sign to the reaction.

This paper uses the reaction because it is a straightforward unifying concept. Care should be taken by the engineer using these methods to be sure the sign of the force matches their intent.

2. FORCES IN STAGNANT SYSTEMS

Newton's Second Law states that the net force acting on a system is equal to its change in momentum per change in time – a stagnant system has no change in momentum so the net force must be zero. Such a system is shown in Figure 1.

The pipe walls are cut by the control surface at B and C. It is not the intent here to determine reactions at each location, but instead to determine overall values for the general reactions R_x and R_y.

There are only a few forces to consider, the first being weight F_W. Also acting on the control volume surface are the pressure forces F_B and F_C. Because there is no flow, these are simply due to hydrostatic pressures. An astute observer may already recognize that only these forces impact the reactions. Nonetheless, it is instructive to consider two pressure forces which are *internal* to the system.

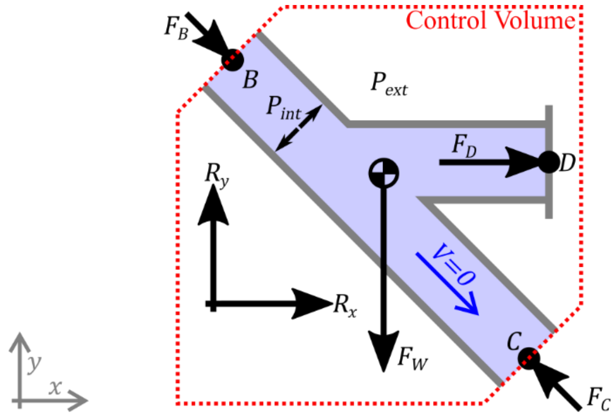

FIGURE 1: FORCES IN A STAGNANT SYSTEM WITH BLIND FLANGE AT POINT D

2.1 Internal Forces

Internal pressure, P_{int}, acts radially outward on the pipe walls. Because the flow is assumed one-dimensional and piping deformation is not considered, the radial pressure does not result in any external reaction.

Internal pressure also acts on non-radial surfaces like the blind flange at point D. Pressure differential at the flange will certainly cause a net force $F_D = (P_{int} - P_{ext}) \times A$. However, the resulting force is taken up by the bolts attaching the flange to the rest of the piping and is therefore internal to the system.

Weight from the horizontal pipe to the flange at point D will also cause a significant moment, resulting in high stresses at the joint to the main piping.

These types of *internal* or *local* forces are of serious concern and *must* be recognized in piping system design. However, they do not influence the external reactions caused by fluid transients of interest for this paper and will not be discussed further.

3. FORCES UNDER STEADY FLOW

Under steady flow, Newton's Second Law no longer indicates that the net force must be zero. Because the flow is steady, the mass within some control volume will be a constant value, but the velocities entering or exiting the control surface can vary. Therefore, Newton's Second Law may take the common form in Equation 1.

$$\vec{F}_{net} = \dot{m}\Delta\vec{V} = \dot{m}(\vec{V}_{out} - \vec{V}_{in}) \tag{1}$$

Note that the inlet and outlet velocities of a constant area system are only equal for incompressible flows. For gas flows, even a straight run will see velocity and thus momentum changes. The examples in Sections 3 and 4 assume incompressible flow to simplify discussion but can easily be adapted to compressible flow.

3.1 Changes In Momentum

The velocity $\vec{V}$ is a vector, so directional changes like flow around a bend will result in a net force, as shown in Figure 2.

Of course, a change in velocity does not require a change in direction, but could also be attained via change in area, like the nozzle shown in Figure 3.

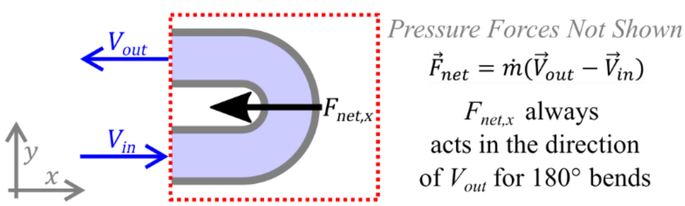

FIGURE 2: NET FORCE FROM FLOW AROUND BEND

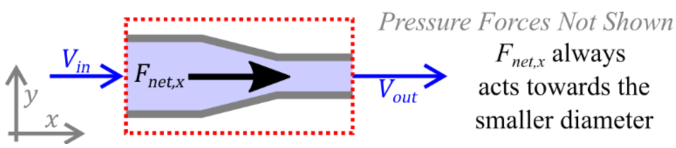

FIGURE 3: NET FORCE FROM FLOW THROUGH REDUCTION

Without reaction forces, the systems in Figures 2 and 3 will move. The net force on either system can be determined either with Equation 1, or by summing all component forces. In these systems, this sum is the reaction plus the net pressure force $(F_{net} = R_x + F_{P,net})$. The reaction is therefore the net pressure force subtracted from the net force as determined by Newton's Second Law $(R_x = \dot{m}\Delta V - F_{P,net})$.

In Figure 2, $\dot{m}\Delta V$ is negative and $F_{P,net}$ is positive, so R_x must also be negative. In Figure 3, $F_{P,net}$ is again positive, but $\dot{m}\Delta V$ is positive. However, the fluid mechanics of the situation mean $F_{P,net}$ is always larger than $\dot{m}\Delta V$, and R_x is again negative.

3.2 Friction

Friction is a shear force opposing fluid flow at fluid/solid interfaces. The diagram in Figure 4 shows a section of a straight run, with a control volume drawn around the fluid surface – only fluid is inside the volume.

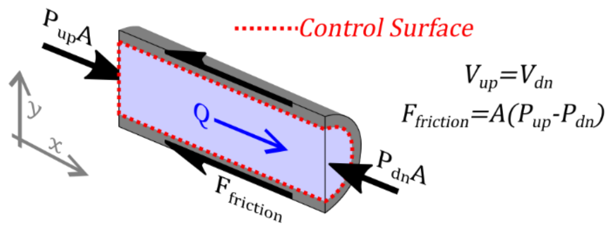

FIGURE 4: CONTROL VOLUME AROUND FLUID ONLY

In Figure 4, the only forces are the upstream and downstream pressure forces and friction. There is no momentum change, so the net force must be zero, meaning the friction and pressure forces counterbalance one another.

A control volume could also be drawn around the piping, in a cylindrical annulus excluding all fluid as in Figure 5.

The only fluid force acting on the piping wall is the friction force, equal and opposite in sign from the control volume in Figure 4. It must therefore be balanced by an external reaction.

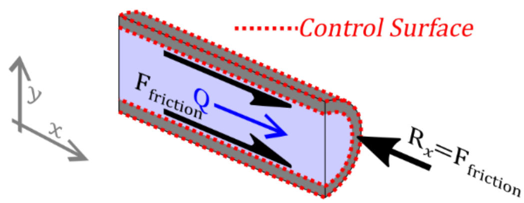

FIGURE 5: CONTROL VOLUME AROUND PIPE ONLY

The reaction force could be determined by first finding the friction force from the pressure differential with a fluid control volume, and then finding the reaction from the friction force with the pipe control volume – a complexity which can be avoided entirely by considering friction an internal force and drawing the control volume to include both piping and fluid as in Figure 6.

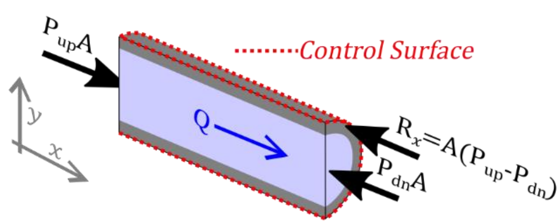

FIGURE 6: CONTROL VOLUME AROUND BOTH PIPE/FLUID

The frictional force does not act on the control volume shown in Figure 6, making the solution straightforward.

3.3 Devices

Complex equipment is, at first glance, difficult to handle. Devices like valves have complex flow paths with a multitude of frictional forces, all in different directions. A centrifugal pump has these issues with the addition of a work term.

Determining these internal effects is extremely difficult. Fortunately, it is not required – the same approach outlined in the previous section can be taken, drawing a control volume around both device and fluid as in Figure 7.

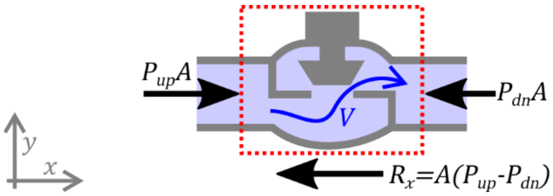

FIGURE 7: CONTROL VOLUME AROUND GLOBE VALVE

As in Figure 6, there are two pressure forces and a reaction force. Additionally, a change in piping diameter like that in Figure 3 is possible, adding a momentum term.

4. STEADY-STATE FORCE ON A PIPING ASSEMBLY

The cases considered so far look at components of a piping system in isolation, which is not always of practical use. A piping assembly may contain direction and area changes, pressure losses due to friction and devices, all of which have been discussed.

As a reminder, the presumed intent of the force analysis is to determine the reactions required to keep the entire system in place. Other internal forces are important but not related to these reactions.

Consider the system in Figure 8. A large diameter upstream pipe makes a right-angle turn at location 1. At location 2, the piping diameter is reduced, causing an increase in flow velocity. The valve at location 3 induces a large pressure loss, and finally the flow exits the system at an angle α above the horizontal at location 4. Straight runs of piping between each location are denoted with H through T. The following two subsections present differing methods for determining the overall reaction on the indicated control volume.

The control surfaces are defined such that they lay on the exterior piping surface and such that flow enters and leaves the system normal to the control surface.

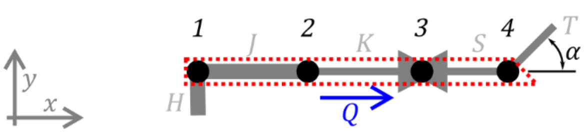

FIGURE 8: A COMPLEX PIPING ASSEMBLY

4.1 Component Reaction Method

One approach is to determine individual internal reactions for each component (1-4 and H-T) and appropriately combine them. For simplicity, the following example considers only R_x, the overall reaction in the x direction.

As indicated by Figure 2, location 1 requires a local reaction dependent on the change in direction of the velocity. There is no flow into the component in the x direction, and the flow out is entirely in x. There is a pressure force acting to the left, from the upstream side of pipe J (P_J^{up}), and an ambient pressure force acting to the right. Rather than handling the ambient pressure force independently, it is easiest to use gauge pressures ($P_{g,J}^{up}$). This makes the local reaction:

$$R_{x,1} = \dot{m}V_J + A_J P_{g,J}^{up} \quad (2)$$

At location 4, there is also a change in direction, but it is not at a right angle. The flow into the component is entirely in x, but flow out is in both x and y. Only velocity in the x direction contributes to the local reaction in x. Similarly, there are two pressure forces acting in in the x direction:

$$R_{x,4} = \dot{m}(V_T \cos\alpha - V_S) - A_S P_{g,S}^{dn} + A_T P_{g,T}^{up} \cos\alpha \quad (3)$$

At location 2, there is an increase in velocity which requires a local reaction as shown by Figure 3.

$$R_{x,2} = \dot{m}\left(V_K - V_J\right) - A_J P_{g,J}^{dn} + A_K P_{g,K}^{up} \quad (4)$$

All pipes have local reactions due to friction. The pressure loss due to friction acts on the flow area, as described by Figure 6. The reaction direction opposes the flow direction. Equation 5 is written for pipe K – similar equations exist for the other pipes.

$$R_{x,K} = -A_K(P_{g,K}^{up} - P_{g,K}^{dn}) \quad (5)$$

Tackling the local reaction for location 3 follows similarly, using the form described by Figure 7.

$$R_{x,3} = -\left(A_K P_{g,K}^{dn} - A_S P_{g,S}^{up}\right) \quad (6)$$

The overall reaction can be determined by adding all these component reactions together.

$$\begin{aligned} R_x = &+\dot{m}V_J + A_J P_{g,J}^{up} \\ &-A_J\left(P_{g,J}^{up} - P_{g,J}^{dn}\right) \\ &+\dot{m}\left(V_K - V_J\right) - A_J P_{g,J}^{dn} + A_K P_{g,K}^{up} \\ &-A_K\left(P_{g,K}^{up} - P_{g,K}^{dn}\right) \\ &-A_K P_{g,K}^{dn} + A_S P_{g,S}^{up} \\ &-A_S\left(P_{g,S}^{up} - P_{g,S}^{dn}\right) \\ &+\dot{m}(V_T \cos\alpha - V_S) - A_S P_{g,S}^{dn} + A_T P_{g,T}^{up} \cos\alpha \end{aligned} \quad (7)$$

There are many repeated terms of opposing sign in Equation 7. This is because many of the individual component control volumes adjoin one another. For example, the valve has a pressure force acting on it from the fluid in the pipe, and the pipe has a fluid force acting on it from the valve. These forces are equal and opposite, so approaching the system piece by piece results in many duplicate calculations. The reaction simplifies to:

$$R_x = \dot{m}V_T \cos\alpha + A_T P_{g,T}^{up} \cos\alpha \quad (8)$$

4.2 Acceleration Reaction Method

Another approach is to consider certain forces internal to the system once more. Rather than approaching each component within the control volume individually, the control volume as a whole can be analyzed directly.

Ignoring everything inside the control volume, flow in at location 1 is vertical and can be completely ignored for the horizontal reaction. There is a change in velocity and pressure force in the x direction at location 4 if α is not $\pm 90°$, which must be counterbalanced.

It is no coincidence that Equation 8 results exactly from this approach. The Component Reaction Method required many steps to arrive at the same result, each with its own opportunity for error. Drawing a control volume around the entire assembly simplifies the process dramatically.

Note that both the momentum term and pressure term depend on the orientation of flow into and out of the assembly. If the inflow and outflow orientations are parallel, no perpendicular force is possible. A common piping configuration is a piping bridge, expansion loop, or other straight run between two right-angle bends (see Figure 9). In such configurations, the steady reaction in the straight run direction must be zero.

5. FORCES UNDER TRANSIENT FLOW

5.1 Transient Flow Fundamentals

It is not intended to discuss in detail the methods of solution for fluid transients in systems here. However, some basics are necessary to understand the impact an acoustic wave has on forces.

Fluid in a piping system propagates pressure waves that travel at acoustic velocity in response to disturbances just like sounds in free air. Fluids in piping systems are generally either liquids or highly compressed gasses, which increases the speed of sound and the energy contained by the wave. The speed of the wave can also be modulated by the flexibility of piping material (especially for liquid transients) and other effects – the actual speed of the wave is called *wavespeed, a*.

Changes in fluid velocity and pressure are strongly coupled. Taking an action like closing a system valve will necessarily change the flow velocity and thus the pressure, a relationship described by the Joukowsky Equation.

$$\Delta P = -\rho a \Delta V \qquad (9 - Joukowsky)$$

The Joukowsky Equation is strictly true only for differential changes but can sometimes be used as an approximation for finite duration changes. Equation 9 and other simple fluid transient estimations do not account for real fluid effects [8]. A numerical simulation based on the fundamental mass, momentum, and energy equations should be used to determine the actual pressures and velocities accurately. Complex but important phenomena such as cavitation and transient friction should be considered. Several such tools exist, many of them based on the Method of Characteristics [9] [10] [11].

The following discussions on force calculation presume an accurate transient flow solution exists. Modifying the underlying flow solution with, for example, differing unsteady friction models will impact the numeric force results, but will not impact the force methodology. Discussion on the fundamental accuracy of the flow results is outside the scope of this paper.

5.2 Approximate Transient Forces

Accurate forces depend on more than just pressure drop. The Endpoint Pressure Method *described in Section 5.2 is common, but it is often inaccurate and is presented here only to simply describe the mechanics of acoustic waves. For example, a steady-state pressure drop due to friction would seem to translate to a steady-state reaction using this approach, but as discussed in Section 4.2, no such force exists.*

Consider a flow of water through a large expansion loop, as in Figure 9. All numerical values required to analyze this system fully in simulation are given in Table 1.

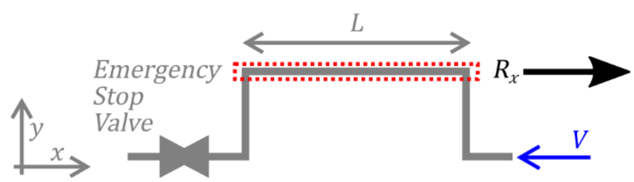

FIGURE 9: EXPANSION LOOP WITH DOWNSTREAM VALVE

Using the Joukowsky Equation, the pressure rise at the valve can be estimated as 323 psi (2.23 MPa). A pressure wave of this magnitude travels backward from the valve through the entire system at the wavespeed, 4800 ft/s (1460 m/s). Along with the pressure wave, there is a velocity wave travelling at the same speed – the fluid upstream of the wave continues flowing at 5 ft/s (1.524 m/s) whereas the fluid downstream of the wave has come to a standstill.

The pressure delta across the loop ($P_{up} - P_{dn}$) is the primary driver of force and will help illustrate the wave's effect on the loop. Figure 10a shows transient pressure traces at the upstream and downstream ends of the loop. As expected, the pressure rise at the downstream end occurs first, as it is closer to the valve. The same pressure rise occurs later at the upstream end.

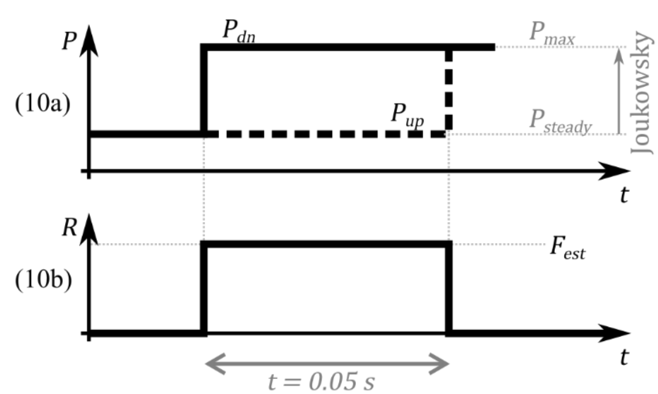

FIGURE 10: BEHAVIOR UNDER INSTANT CLOSURE

The force can be estimated by looking only at the endpoint pressures and the flow area. This method is termed here the Endpoint Pressure Method. For this example, the estimated maximum force, $F_{est} = \Delta P \times A$, is about 36,600 lbf (163 kN). Figure 10b shows the behavior of the force over time.

TABLE 1: NUMERICAL INPUTS & RESULTS FOR FIGURE 9

	Value	USC Units	SI Units
INPUTS	Inner Diameter	12.00 in	30.48 cm
	Friction Factor	0	0
	Length L	240.0 ft	73.15 m
	Initial Velocity	5.000 ft/s	1.524 m/s
	Wavespeed	4,800 ft/s	1,460 m/s
	Fluid Density	62.40 lbm/ft^3	1,000 kg/m^3
	Valve Close Time	0 s	0 s
RESULTS	Joukowsky Pressure Rise	323.2 psi	2.228 MPa
	Estimated Maximum Force	36,560 lbf	162.6 kN
	Force Duration	0.05 s	0.05 s

If the valve does not close instantly, the behavior is different. The pressure at any given point will not rise instantly but will be a function of the valve position vs. time. Assuming the valve instead closes such that flow decreases linearly over 0.1 seconds, the wave will be 480 feet (146 meters) long. The behavior now appears as in Figure 11.

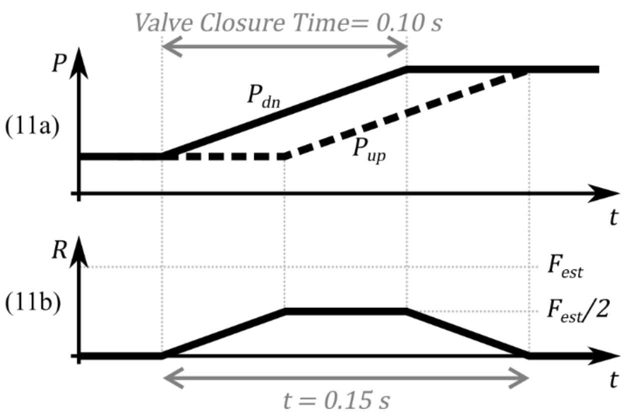

FIGURE 11: BEHAVIOR UNDER NON-INSTANT CLOSURE

Only half of the wave can exist inside the loop at any given time, meaning the maximum force has been reduced by half. The duration of maximum loading is the same, but the duration of any loading is tripled.

A characteristic ratio (Equation 10) can be defined from the speed of generation of the transient, the wavespeed, and the length of the piping assembly in question.

$$C_L \equiv \frac{L}{a\Delta t} \tag{10}$$

If C_L is greater than one, it indicates that the entire wave can fit inside the assembly, and the force will be the maximum value. If it is less than one, it reports the fraction of the wave within the assembly, and the force will be attenuated.

C_L is a simplified metric that neglects real effects, but the general concept is sound for liquid flows. For gas flows, the width of the wave can compress and expand, which is discussed briefly in Section 7.

5.3 Complete Transient Reactions with the Acceleration Reaction Method

Calculating transient reactions with the Endpoint Pressure Method, as touched on in Section 5.2, neglects the effects of momentum entering and exiting the system. Even if these momentum terms are included, there is no true accounting of internal transient effects. Taking the upstream and downstream pressures and velocities at some given time and using those to calculate a force is a steady-state approach that completely neglects the internal effects.

Instead, Newton's Second Law should be applied directly to the system, without unnecessary simplification (Equation 11).

$$\vec{F}_{net} = \frac{d\overrightarrow{mV}}{dt} = \frac{d}{dt}\int \rho\vec{V}d\mathbb{V} \tag{11}$$

The net force is also equal to the summation of applied forces. This includes all external forces acting on the control volume, such as weight or pressure forces at a fluid boundary, and the reaction (Equation 12).

$$\vec{F}_{net} = \sum \vec{F}_{applied} + \vec{R} \tag{12}$$

With Reynolds Transport Theorem, Equation 11 can be expanded to Equation 13.

$$\vec{F}_{net} = \int_{C\mathbb{V}} \frac{\partial}{\partial t}\rho\vec{V}d\mathbb{V} + \oint_{CS} \rho\vec{V}\left(\vec{V}\cdot\hat{n}\right)dA \tag{13}$$

The control volume is chosen specifically so that any flow entering or leaving intersects it normal to the control surface, which means the flux integral can be simplified as in Equation 14, where ϕ is positive one for outflows and negative one for inflows. Note that the right-hand side of Equation 14 is effectively the same as the right-hand-side of Equation 1.

$$\oint_{CS} \rho\vec{V}\left(\vec{V}\cdot\hat{n}\right)dA = \sum(\phi\rho VA)\vec{V} = \sum \phi\dot{m}\vec{V} \tag{14}$$

With $d\mathbb{V} = Adx$ and $\rho AV = \dot{m}$, the transient term of Equation 13 can be modified as shown in Equation 15.

$$\int_{C\mathbb{V}} \frac{\partial}{\partial t}\rho\vec{V}d\mathbb{V} = \int_{C\mathbb{V}} \frac{\partial\vec{\dot{m}}}{\partial t}dx \tag{15}$$

Simplification of this term is more difficult and cannot be done without introducing discretization error. To simplify further, several actions are taken. First, consider only a single pipe of length L. Second, recognize that the velocity in each pipe is colinear with the pipe axis, and thus the contribution of Equation 15 to the reaction is also colinear with the pipe. Finally, in typical systems, the fluid volume within piping far exceeds the volume within devices (e.g., valves or pumps) – in such cases it is safe to treat the transient term as negligible and disregard it. Therefore, each of M straight runs can be treated independently and summed together via Equation 16.

$$\int_{C\mathbb{V}} \frac{\partial \vec{\dot{m}}}{\partial t} dx = \sum_{m=1}^{M_{pipes}} \int_0^L \frac{\partial \vec{\dot{m}}}{\partial t} dx \tag{16}$$

Of course, there are not an infinite number of values available, but only some discrete set of N nodes. The integral can be split into an integral for each of these nodes via Equation 17.

$$\sum_{m=1}^{M_{pipes}} \int_0^L \frac{\partial \vec{\dot{m}}}{\partial t} dx = \sum_{m=1}^{M_{pipes}} \sum_{n=1}^{N_{nodes}} \int_0^{\delta x} \frac{\partial \vec{\dot{m}}}{\partial t} dx \tag{17}$$

The integral in Equation 17 must be evaluated. For a given step in time, the mass flow in each section can be approximated as a fixed value, averaged from the neighboring points. Averaging mass flow makes the integrand no longer a function of x so it can be taken out of the integral. The temporal derivative also needs to be discretized (Equation 18).

$$\sum_{m=1}^{M_{pipes}} \sum_{n=1}^{N_{nodes}} \int_0^{\delta x} \frac{\partial \vec{\dot{m}}}{\partial t} dx \approx \sum_{m=1}^{M_{pipes}} \sum_{n=1}^{N_{nodes}} \frac{\Delta \vec{\bar{\dot{m}}}}{\Delta t} \Delta x \tag{18}$$

The discretization step is the only significant approximation, and the error is directly dependent on the degree of the discretization. The final equation for the reaction is Equation 19.

$$\vec{R} \approx \sum_{m=1}^{M_{pipes}} \sum_{n=1}^{N_{nodes}} \frac{\Delta \vec{\bar{\dot{m}}}}{\Delta t} \Delta x + \sum \phi \dot{m} \vec{V} - \sum \vec{F}_{applied} \tag{19}$$

In the special, but common, case of a system such as that shown in Figure 9 with flow entering and exiting 90 degrees from a colinear arrangement of piping, the reaction in the colinear direction will be simplified to only the first term of Equation 19. In stark contrast to the Endpoint Pressure Method described in Section 5.2, this equation would have *no* pressure terms.

On close inspection, the first term bears a strong resemblance to $F = ma$, which should not come as a surprise – Equation 19 is nothing more than a representation of Newton's Second Law, specifically formulated for transient analysis of fluid systems.

The *Acceleration Reaction Method* is so named because this first term is the summation of mass times acceleration values at each discrete point.

5.4 Complete Transient Reactions with the Component Reaction Method

The approach taken for a steady reaction in Section 4.1 can also be taken for the transient reaction. However, Equation 7 cannot be used directly because it does not account for internal transient effects.

Looking back at Figure 5, it is clear that flow within the pipe creates a frictional force that must be opposed by a component reaction. Under general transient flow, different areas of the pipe will see different velocities and thus different frictional forces.

Accounting for this requires discretization just like the one performed in Equation 18. Determining the effect of friction during a transient is not as simple as looking at the pressure drop. Instead, a resistance $\mathbb{R}$ is used to characterize friction, which is determined from appropriate transient values for a particular node.

$$\mathbb{R} \equiv \frac{f \Delta x}{2\rho D A^2} = \frac{\Delta P_{friction}}{\dot{m}^2} \tag{20}$$

Equation 5 for a single pipe then becomes Equation 21 under transient conditions. The absolute value on $\dot{m}$ is used to ensure the sign of the reaction is correct for both flow directions.

$$R_{friction} \approx - \sum_{n=1}^{N_{nodes}} A \mathbb{R} |\bar{\dot{m}}| \, \bar{\dot{m}} \tag{21}$$

When replacing the frictional terms in Equation 7 with Equation 21, *none* of the pressure terms cancel, making the result much more unwieldy than Equation 19. Nonetheless, it is possible to take this approach, summing individual pressure and momentum terms along with the friction terms from Equation 21 – the *Component Reaction Method*.

Note that the friction factor f and density ρ are not in general constant, and $\mathbb{R}$ must be updated for each node every time step.

5.5 A Note on Discretization and Accuracy

Both Equation 7 with Equation 21 substituted in, and Equation 19 will – given enough computation sections – return the same true result. Due to the differences in the methods, coarse discretizations will show minor differences.

It should also be noted that coarse discretizations will not be able to track the wave or its shape as well as finer discretizations, meaning the force results will be lower in accuracy. For example, if the simple case described by Figure 9 is analyzed with many numerical nodes, the square wave depicted by Figure 10 results. However, as the number of nodes decreases, the waveform will appear to change shape as in Figure 12. In this instance, the maximum force is still predicted correctly, but this is not always the case.

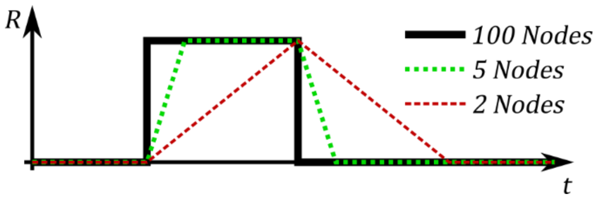

FIGURE 12: EFFECT OF DISCRETIZATION ON REACTIONS

6. NOTABLE IMPACTS VS TRADITIONAL RESULTS

The effects of friction, momentum, and complexities of inline equipment become more evident when directly comparing the behavior of simplified methods and Equation 19. The simple method used for comparison here is the Endpoint Pressure Method (Section 5.2).

The examples in Section 6 are all based on Figure 9/Table 1. In all cases, a surge of pressure is generated by instantly halting a flow of equal velocity. Therefore, it seems reasonable to calculate a theoretical *Joukowsky Maximum* (with Equation 9 pressure rise and flow area) that is equal for every case.

For a summary of inputs and results, see Tables 2 & 3.

6.1 Friction

As a baseline, it is useful to consider a frictionless case. If there are no pressure losses, the Joukowsky Maximum, Endpoint Pressure Method, and Equation 19 all correctly determine the maximum force, as seen in Figure 13. With no friction or momentum flux terms, the endpoint pressures are, in fact, the only forces acting on the assembly.

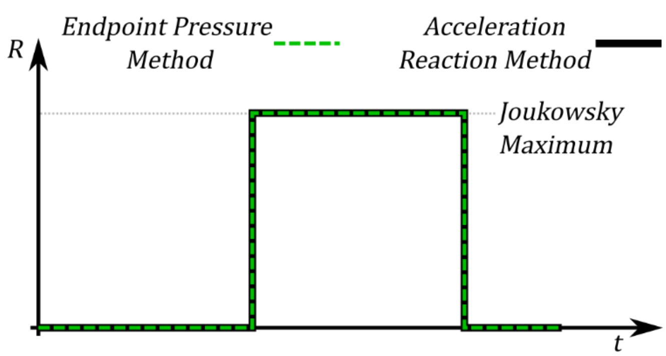

FIGURE 13: COMPARISONS FOR FRICTIONLESS FLOW

With friction, the behavior is noticeably different, as indicated by Figure 14.

First, there is a steady-state pressure drop – the upstream pressure will be higher than the downstream pressure. Therefore, the Endpoint Pressure Method indicates an incorrect steady-state reaction in the negative (downstream) direction.

Second, internal pressure continues to rise after the initial surge under frictional flow, a phenomenon known as line pack [8] or recovery of pressure loss due to friction.

Once the wave enters the assembly, the higher downstream pressure forces the piping in the downstream direction, requiring a reaction in the upstream direction. However, pressure is not the only source of force acting on the piping. Upstream of the wave, flow continues at the original velocity and imparts a frictional force on the pipe. The friction force *also* acts in the downstream direction, increasing the required reaction. The contribution of friction to the reaction will decrease as the wave moves and more fluid comes to a halt.

The Endpoint Pressure Method will instead interpret the losses due to friction as *lowering* the reaction. The comparison is shown in Figure 14. The impact of friction is generally small and is exaggerated in Figure 14, but it can be significant in long pipelines or in highly viscous flows.

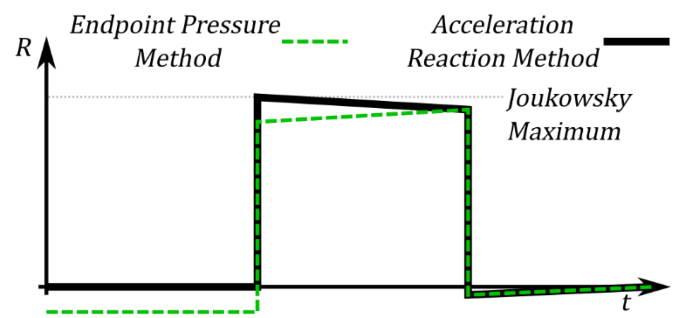

FIGURE 14: COMPARISONS FOR FRICTIONAL FLOW

6.2 Inline Losses

If a static loss, such as an orifice, was placed midway through the assembly, there may be significant pressure loss in steady-state. Such a device has no significant impact on transient endpoint pressures until well after the wave enters the system.

The steady-state pressure loss problem exists here as well, but it is due to the loss at the device instead of in the piping. The Endpoint Pressure Method neglects the reaction at the device, so the overall reaction appears unchanged when the wave passes through the device. The result is similar to the previous case, but the impact on the forces plotted in Figure 15 is, perhaps, surprising.

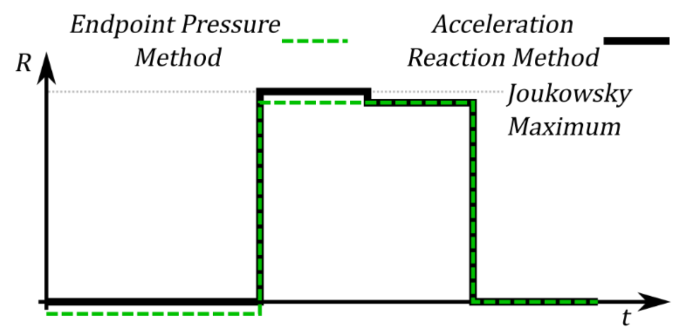

FIGURE 15: COMPARISONS FOR AN INLINE LOSS

6.3 Inline Closure

The instantly closing emergency stop valve is moved from its downstream location to the midpoint of the control volume, as in Figure 16. Frictional losses through the valve and piping are negligible in steady-state.

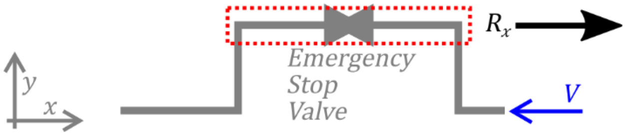

FIGURE 16: CONFIGURATION FOR INLINE CLOSURE

Figure 17 shows Equation 19 calculating a maximum reaction double that of the apparent Joukowsky Maximum. When the valve closes, it propagates an *increase* in pressure upstream, which is paired with the propagation of an equivalent *decrease* in pressure downstream.

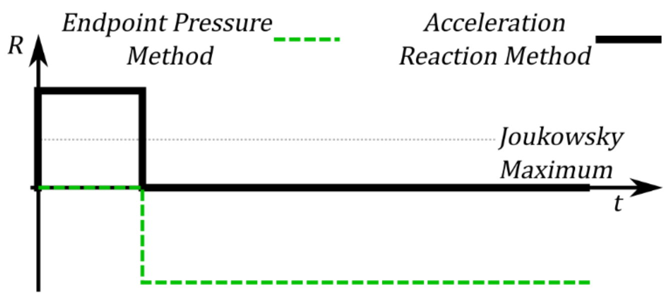

FIGURE 17: COMPARISONS FOR INLINE VALVE CLOSURE

Failing to capture the maximum force is not an error with the Joukowsky equation, but rather a misapplication. The Joukowsky equation can correctly predict the change in pressure on both sides of the valve, but the reaction depends on the overall pressure difference – not that generated by a single wave.

In this simple example it is easy to recognize that there are two waves of opposite signs and the force must be doubled. However, taking this approach to wave analysis is not recommended in realistic systems with complex behaviors. Attempting to determine how to modify the rule quickly becomes more overwhelming. It is easy to get confused applying the Joukowsky equation and great care should be taken using it to validate results – if it is even used at all.

An additional, more shocking, result occurs with the Endpoint Pressure Method. While it correctly determines the magnitude of the force, the sign is not correct, the timing is wrong, and the force seems to be maintained indefinitely.

The instant closure of the valve propagates two waves, which both serve to strongly accelerate the local fluid. These accelerations contribute force in a common direction. After the waves exit the system, the system becomes stagnant – there is nothing left to generate force, so the reaction drops to zero.

The Endpoint Pressure Method, however, is completely unaware of the pressure waves *until* they reach the endpoints. The sudden pressure differential, with the upstream pressure larger than the downstream pressure, will appear to pull the system in the upstream direction, requiring a negative reaction.

6.4 Inline Reduction

Finally, consider the case where the instantly closing valve is again downstream, and a reducer exists at the assembly midpoint, as in Figure 18. The downstream velocity remains the same as in the other cases – the upstream piping is larger diameter making the upstream velocity lower.

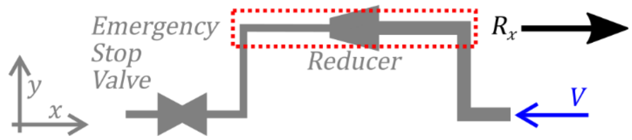

FIGURE 18: CONFIGURATION FOR INLINE REDUCTION

Figure 19 shows that the Endpoint Pressure Method dramatically misestimates the steady-state force, as not only is the upstream static pressure higher, but it acts on a larger area. In fact, the area difference means the reaction is predicted as negative for the entire transient, even though the true reaction is only ever zero or positive.

Moreover, the Endpoint Pressure Method does not account for the change in momentum across the reduction. The reducer sees a positive local reaction (see Section 3.1), but acceleration of flow through the reducer reduces the reaction. This reduction in reaction is present until the wave passes through the reducer.

Again, the Joukowsky Maximum seems to be violated. The Joukowsky approach assumed a constant flow area associated with the downstream velocity. However, the downstream pressure rise cannot simply be applied to the upstream area.

Due to the nature of acoustic transients, the reducer will partially reflect the wave, even if it is lossless. The velocity will not stay at zero, but becomes negative, inducing a momentum effect not only at the reducer, but within two waves travelling in opposite directions as in the previous section.

Accounting for these effects with the Endpoint Pressure Method or Joukowsky Equation is not straightforward.

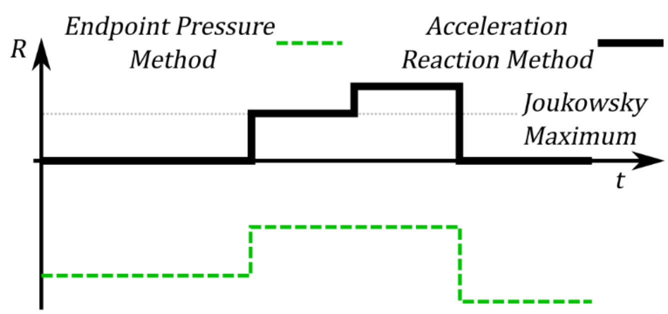

FIGURE 19: COMPARISONS FOR FLOW AREA REDUCTION

6.5 Notes On Simplified Methods

It is important to state with full clarity that the Joukowsky Equation and Endpoint Pressure Method for calculating forces are not incorrect so much as they are incomplete. It would not be difficult to mend either method in any of the above cases with special rules or adjustments in given cases. However, this becomes exceptionally difficult when complex, but common, situations arise.

For example, a given assembly could have multiple changes in pipe diameter and a high loss valve that closes rapidly but not fully. It could also have connections to neighboring piping that are not at a convenient right angle. The system in which it is operating may see several simultaneous pressure waves from remote events, including events like column separation.

Calculating forces with Equation 19 may be difficult by hand, but compared to complex patchworked rules, it is easier overall and far less likely to fall victim to easily made mistakes.

TABLE 2: INPUTS TO SECTION 6 EXAMPLES, AS MODIFIED FROM FIGURE 9

Figure 10/13	Frictionless, instant closure
Figure 14	Heavy friction – 50x normal friction from an absolute roughness of 0.0018 in
Figure 15	Device with K=100 inserted at midpoint of *L*
Figure 16/17	Instantly closing valve (Cv=20,000) inserted at midpoint of *L*
Figure 18/19	Reducer inserted at midpoint of *L*. Upstream inside diameter changed to 16 in (40.64 cm)

TABLE 3: SELECTED RESULTS FROM SECTION 6 EXAMPLES

Figure	Pressure Method Maximum	Pressure Method Amplitude*	True Maximum†	True Amplitude*†
10/13	36,556 lbf 162.61 kN	36,574 lbf 162.69 kN	36,518 lbf 162.44 kN	36,518 lbf 162.44 kN
14	33,954 lbf 151.03 kN	37,425 lbf 166.47 kN	36,503 lbf 162.37 kN	37,341 lbf 166.10 kN
15	34,653 lbf 154.14 kN	36,557 lbf 162.61 kN	36,510 lbf 162.40 kN	36,510 lbf 162.40 kN
16/17	-73,262 lbf -325.89 kN	73,262 lbf 325.89 kN	73,176 lbf 325.50 kN	73,194 lbf 325.58 kN
18/19	-51,563 lbf -229.36 kN	56,927 lbf 253.22 kN	56,295 lbf 250.41 kN	56,295 lbf 250.41 kN

NOTE Joukowsky Maximum is 36,555 lbf (162.61 kN) in all cases.
* Amplitude is defined here as the difference between the overall Maximum minus overall Minimum.
† The True values were calculated with 100 nodes using the liquid Method of Characteristics solver AFT Impulse [11].

7. NOTES ON GAS FLOW

The Acceleration Reaction Method (Equation 19) does not assume anything about the nature of the fluid, and it is equally valid for both liquid and gas flows. The same could be said about the Component Reaction Method (Equations 7 and 21), although determining accurate frictional values in transient compressible flow for force calculation is challenging.

The behavior of gas transients differs substantially from liquid transients. A liquid transient analysis generally assumes an isothermal, constant density, constant wavespeed flow. In gas transients, none of these are good assumptions. As gas becomes compressed, it will heat up, which in turn drives up the speed of sound. For gas flow in the case described by Figure 9, with a non-instant downstream valve closure, the front of the wave will have a lower speed of sound than the back of the wave. Therefore, the amount of the wave that can be contained within the loop depends not only on the length of the loop, but its distance from the valve. The impact of gas dynamics on transient loads is discussed in detail in [6].

8. CONCLUSION

The authors have seen numerous erroneous reaction calculations. Often, the incomplete Endpoint Pressure Method is used due to its apparent simplicity. To be emphatically clear: the Endpoint Pressure Method is *very often incorrect* in realistic situations, and the authors *do not* recommend its use under any circumstances.

The complete Acceleration Reaction Method presented here is not excessively difficult to utilize if the results of a transient fluid simulation are available. Instead of speculating on the importance of the correct force, the fundamental approach is recommended for any liquid or gas piping system.

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