# When the Joukowsky Equation Does Not Predict Maximum Water Hammer Pressures

Proceedings of the ASME 2018 Pressure Vessels and Piping Conference
PVP2018
July 15-20, 2018, Prague, Czech Republic

PVP2018-84050

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Pressure Vessels & Piping Conference

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# WHEN THE JOUKOWSKY EQUATION DOES NOT PREDICT MAXIMUM WATER HAMMER PRESSURES

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## ABSTRACT

The Joukowsky equation has been used as a first approximation for more than a century to estimate water hammer pressure surges. However, this practice may provide incorrect, non-conservative, pressure calculations under several conditions. These conditions are typically described throughout fluid transient text books, but a consolidation of these issues in a brief paper seems warranted to prevent calculation errors in practice and to also provide a brief understanding of the limits and complexities of water hammer equations.

To this end, various issues are discussed here that result in the calculation of pressures greater than those predicted by the Joukowsky equation. These conditions include reflected waves at tees, changes in piping diameter, and changes in pipe wall material, as well as frictional effects referred to as line pack, and the effects due to the collapse of vapor pockets. In short, the fundamental goal here is to alert practicing engineers of the cautions that should be applied when using the Joukowsky equation as a first approximation of fluid transient pressures.

## KEY WORDS

Water hammer, fluid transient, line pack, reflected pressure waves, cavitation, vapor collapse, liquid column separation.

## NOMENCLATURE AND SYMBOLS

| | |
|---|---|
| $a$ | wave speed, m/s (ft/s) |
| $A$ | pipe cross-sectional area, m$^2$ (ft$^2$) |
| $Cv$ | valve flow coefficient, $\text{gpm}/\sqrt{\text{psid}}$ |
| $D$ | pipe inside diameter, m (ft) |
| $f$ | friction factor (Darcy/Moody) |
| $g$ | gravitational acceleration, 9.81 m/s$^2$ (32.174 ft/s$^2$) |
| $H$ | piezometric head, m (ft) |
| $\Delta H_J$ | piezometric head change, Joukowsky Eq. 1, m (ft) |
| $P$ | pressure, kPa (psi) |
| $\Delta P_{fr}$ | friction recovery pressure, kPa (psi) |
| $\Delta P_J$ | pressure change, Joukowsky, Eq. 3, kPa (psi) |
| $\Delta P_{max}$ | maximum total pressure, kPa (psi) |
| $Q$ | volumetric flow rate, m$^3$/s (ft$^3$/s) |
| $V$ | velocity, m/s (ft/s) |
| $\rho$ | liquid density, kg/m$^3$ (lbm/ft$^3$) |

## ABBREVIATIONS

| | |
|---|---|
| DGCM | Discrete Gas Cavity Model |
| DVCM | Discrete Vapor Cavity Model |
| MOC | Method of Characteristics |

## INTRODUCTION

The rigorous study of water hammer reaches back into the 19$^{th}$ century (Bergant et al. [1], Ghidaoui et al. [2]). Among the excellent text books on water hammer are those of Thorley [3], Wylie and Streeter [4], Swaffield and Boldy [5], Leishear [6], and Chaudhry [7].

The field of water hammer is well established in academia as well as in industry, where industry is often tasked with designing complicated piping systems sometimes many kilometers in length. Further, engineers in industry are typically under budgetary and schedule constraints and often need to make decisions with sometimes incomplete and imperfect information. As a result, in many cases engineers in industry rely on quick, handbook formulas to make decisions based on estimates. One such powerful and important formula for water hammer is usually credited to Joukowsky [8] and is therefore often called the "Joukowsky equation". Other names that one finds in industry for this equation are, in various forms, the "Basic Water Hammer Equation", the "Instantaneous Water Hammer Equation" and the "Maximum Theoretical Water Hammer Equation". Note that research in recent years showed that researchers prior to Joukowsky discovered this same equation, but Joukowsky's name is most often associated with this equation. For more on the history of water hammer, consult Tijsseling and Anderson [9].

Just two decades ago water hammer was still largely considered by industry to be a niche specialty. As a result, water hammer studies were often outsourced to specialized and experienced consultants. In the last two decades, there has been a significant growth in the availability of user-oriented (i.e., graphically based and menu-driven) commercial software for water hammer simulation (Ghidaoui et al. [2]).

While there is certainly an overall appreciation for the water hammer phenomenon in industry, mistakes are easily made when using the simplified Joukowsky equation. This equation can be mistakenly misunderstood in industry to be a worst case, conservative equation. A clear understanding is demanded with respect to the situations where non-conservative pressure estimates are obtained when using this equation.

Academic papers acknowledge that the Joukowsky equation is not always conservative, but that knowledge does not always make its way into industrial applications. The purpose of this paper is to identify for the practicing engineer those situations where the Joukowsky equation does not provide worst case, conservative pressure predictions. This knowledge should result in safer and more cost-effective piping systems.

## THE JOUKOWSKY EQUATION

The Joukowsky equation relates the increase in piezometric head or pressure resulting from an instantaneous reduction in velocity (often conceptualized as an instant valve closure). Water hammer theory historically started under the purview of civil engineers for large-scale water works projects. As such, Joukowsky presented his equation in terms of piezometric head (e.g., see Thorley [3], Wylie and Streeter [4], Swaffield and Boldy [5] and Chaudhry [7]):

$$\Delta H_J = -a\Delta V/g \tag{1}$$

The fundamental relationship between head change and pressure change is given by:

$$\Delta P = \rho g \Delta H \tag{2}$$

Hence the Joukowsky equation 1 can also be written in a form more frequently used by mechanical engineers (Leishear [6]):

$$\Delta P_J = -\rho a \Delta V \tag{3}$$

where $\Delta P_J$ equals the pressure change due to a fluid transient (note that some call this the potential surge), $\rho$ is the density, $a$ is the wave speed (also known as the celerity), and $\Delta V$ is a change in fluid velocity. Equations 1 and 3 are both equally valid equations to communicate the instantaneous velocity reduction principle.

An important parameter in Eqs. 1 and 3 is the wave speed, "$a$". The wave speed expresses the propagation velocity of a pressure wave in a pipe. It is less than the liquid acoustic velocity (speed of sound in the unconfined liquid) and depends on the pipe material and liquid in the pipe, as well as on the external pipe supports and piping dimensions. It can be predicted with modest accuracy by equations developed in the literature and can also be measured in installed systems. Consult the previously cited texts for more information on wave speed and analytical prediction methods. In zero-g systems (such as those found in some space system applications), Eq. 1 is undefined when dividing by zero g. The Eq. 3 formulation retains applicability in such cases. This topic is explored in more depth in Walters [10].

Throughout this paper the authors refer to both Eqs. 1 and 3 interchangeably, where these two equations are understood to be essentially equivalent.

### Unappreciated Limitations of the Joukowsky Equation

Undoubtedly, Eq. 1 was a significant contribution to the practice of piping engineering at the turn of the 20$^{th}$ century. Even at that time, many of the limitations to applying Eq. 1 were discussed. However, practicing engineers may be unaware of these limitations, and engineering handbooks often gloss over these limitations for brevity.

In principal, Eq. 1 only claims validity the moment after the velocity decrease (e.g., valve closure). However, practicing engineers often apply it as if it retains validity both immediately after the velocity decrease/valve closure as well as at all times thereafter, assuming that no other independent transients occur.

Since Eq. 1 is often applied in this manner, the limitations of this equation are discussed with respect to its validity after the initial transient occurs. These limitations are as follows:

- Straight, constant diameter piping of uniform material, wall thickness, and structural restraints
- Uniform pipe friction
- Minimal friction pressure drop in piping (explained in a later section)
- Minimal fluid-structure interaction with the piping and supports
- No cavitation or gas release
- No trapped, or entrained, gases in the piping (i.e., it is 100% full of liquid)
- No external heat transfer that can change any of the piping and fluid physical properties or cause phase changes
- Constant liquid density and constant bulk modulus
- One-dimensional fluid flow
- Linearly elastic piping material

### Example 1: Joukowsky Equation Calculation

To introduce the equations, consider an example that applies Eqs. 1 and 3. This example is adapted from Chaudhry ([7] p. 10). Compute the conditions in a 0.5 m (1.64 ft) diameter pipe conveying oil. Determine the pressure increase if the steady volumetric flow rate of 0.4 m$^3$/s (14.1 ft$^3$/s) is instantaneously stopped at the downstream end. Assume the oil density is 900 kg/m$^3$ (56.2 lbm/ft$^3$) and the wave speed is 1,291 m/s (4,236 ft/s).

$$A = \pi D^2/4 = 0.196 \text{ m}^2 \ (2.11 \text{ ft}^2) \tag{4}$$

$$\Delta V = \Delta Q/A = -2.04 \text{ m/s} \ (-6.68 \text{ ft/s}) \tag{5}$$

From Eq. 1:

$$\begin{aligned}\Delta H_J &= -a\Delta V/g = -(1291 \text{ m/s})(-2.04 \text{ m/s}) / (9.81 \text{ m/s}^2)\\ &= 268 \text{ m } (880 \text{ ft})\end{aligned} \tag{6}$$

From Eq. 3:

$$\begin{aligned}\Delta P_J &= -\rho a \Delta V = -(900 \text{ kg/m}^3)(1291 \text{ m/s})(-2.04 \text{ m/s})\\ &= 2{,}367 \text{ kPa } (343 \text{ psi})\end{aligned} \tag{7}$$

Note that this is the pressure increase due to an instantaneous velocity decrease at a downstream valve. To obtain the peak pressure at the valve, the pressure change must be added to the pre-existing, steady-state, static pressure.

### Exploring Joukowsky Equation Limitations

To understand the conditions when Eq. 3 can be exceeded, test or field data is referenced and is reinforced with analytical explanations and solutions when available. Finally, numerical simulations are used to explore different conditions.

The numerical simulation tool used in this study is referenced in Ghidaoui et al. [2] and is commercially available (see Applied Flow Technology [11]). This software uses the widely accepted Method of Characteristics (MOC). It includes both the Discrete Vapor Cavity Model (DVCM) and the Discrete Gas Cavity Model (DGCM) for modeling transient cavitation and liquid column separation. For more information on the DVCM and DGCM, see Bergant et al. [1] and Wylie and Streeter [4]. Considerations of frequency-dependent friction and damping due to fluid-structure interaction (FSI) are not included in the numerical simulations.

Three applications where Eq. 3 may not be conservative are:

1. Transient cavitation and liquid column separation
2. Line pack
3. Piping system reflections (networks, components, area changes and surge suppression devices)

## TRANSIENT CAVITATION AND LIQUID COLUMN SEPARATION

When a negative transient pressure wave reduces the local pressure in the piping system to the vapor pressure of the liquid, vapor is generated. The terminology covering this area is not completely consistent in the literature. Liquid column separation can be considered to exist when the vapor volume is such that it encompasses the entire cross section of the pipe. Hence, the continuous "column" of liquid is no longer intact and it separates.

A more modest situation occurs when the vapor is either smaller in volume and/or distributes itself along a length of pipe such that the liquid stays more or less intact. A bubbly portion of the liquid in the piping exists but may not encompass the entire pipe cross section.

It is not the purpose of this paper to explain the details of this complex phenomenon. Consult Bergant et al. [1] for more information and references on this important aspect of water hammer. For the purposes of this paper we will refer to this entire situation as *transient cavitation*. From an analytical point of view, it is well known that

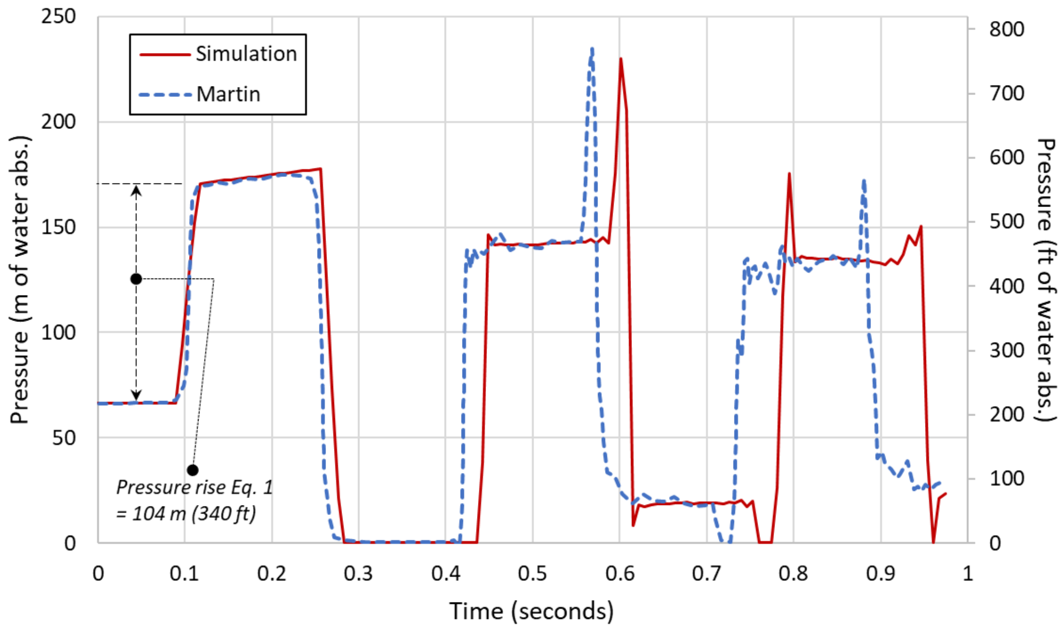


**Figure 1: Example 2 - Experimental and numerical predictions of pressures during transient cavitation, compared to the maximum predicted Joukowsky pressure (Eq. 1)**

predicting water hammer behavior when transient cavitation is occurring, or has recently occurred, is extremely difficult. The examples presented here demonstrate this point.

Bergant et al. [1] reported that Joukowsky himself was “the first to observe and understand column separation”. When transient cavitation occurs, the normal mechanism of water hammer wave propagation is disrupted. It is possible for liquid velocities to become larger than the original velocity, and the wave reflection processes in the liquid phase (Wiley and Streeter [4] and Leishear [6]) become exceedingly more complex. In short, when a cavity collapses, the pressure increase can be much higher than Eq. 3 predicts.

## Example 2: Vapor Collapse

Martin ([12], p. 86, Fig. 6, $x/L = 1$) presents experimental evidence of pressures rising above those predicted by Eq. 1 following transient cavitation. Fig. 1 shows Martin’s data plotted against a numerical simulation that used the DGCM in Applied Flow Technology [11]. The simulation in Fig. 1 accounted for the varying supply pressure as reported by Bergant et al. [1, Fig. 4]. The Eq. 1 pressure rise is predicted to be 104 m (340 ft) of water resulting in a maximum pressure of 171 m (560 ft) near 0.1 seconds. Cavitation begins at this location near 0.3 seconds. Both experiment and simulation in Fig. 1 show a peak pressure of about 235 m (769 ft) near 0.6 seconds. A simulation using constant supply pressure (not shown) produces similar results to Fig. 1. This data indicates pressures can exceed Eq. 1 when transient cavitation occurs.

The simulation assumed a wavespeed of 1,230 m/s (4,035 ft/s), a Darcy friction factor of 0.031 and used 12 computing sections. The valve was modeled as a linear decrease in flow over 25 ms. The maximum vapor size was predicted to be 1.6 cm$^3$ (0.27% of the computing volume).

## Practical Vapor Collapse Advice for Engineers

Check the results to determine if the negative pressure from Eq. 3 subtracted from the steady-state operating pressure drops below the vapor pressure. This occurs either downstream of a valve immediately after closure or upstream of the valve after the wave reflection and the negative wave arrives. If so, then Eq. 3 may not yield a worst case, conservative, maximum pressure. A more detailed numerical simulation should be considered. Fig. 2 helps understand this statement. In Fig. 2 at left, the sum of the operating pressure and the negative Joukowsky Eq. 3 is above the vapor pressure and cavitation is not possible. In Fig. 2 at right, the sum of the operating pressure and the negative Joukowsky Eq. 3 is below the vapor pressure and cavitation is then possible.

Changes in elevation also lead to conditions that cause the formation of vapor pockets and vapor collapse. Note that cavitation may occur at high points in the piping, where flow separation occurs.

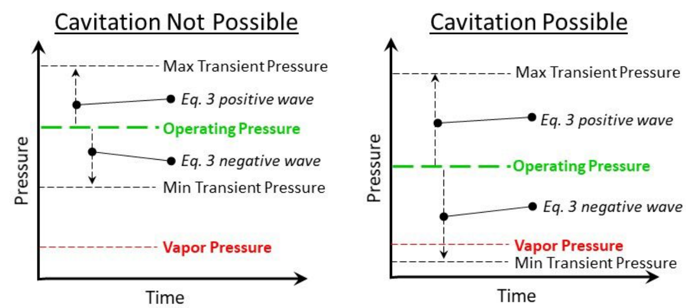


**Figure 2: Diagram to help determine the possibility of transient cavitation**

## LINE PACK

The phenomenon of line pack is another complexity that occurs during fluid transients. It is not the purpose of this paper to explore line pack in full detail. Liou [13] offers a detailed discussion of line pack and a new, powerful method for predicting the peak pressure resulting from a combination of line pack and Eq. 1 (often called the “potential surge” in the context of line pack). Additional discussion can also be found in Thorley [3], Wylie and Streeter [4], Chaudhry [7] and Kaplan et al. [14].

Liou’s introductory paragraph [13] offers an excellent summary of line pack:

> “In pipeline transients, frictional resistance to flow generates line packing, which is a sustained pressure increase in the pipeline behind the water hammer wave front after the closure of a discharge valve. This phenomenon is of interest to cross-country oil pipelines and long water transmission mains because the sustained pressure increase can be very significant relative to the initial sudden pressure increase by water hammer and can result in unacceptable overpressures”.

Line pack is most dramatic when frictional pressure drop is significant. As Liou suggests, line pack is often associated with highly viscous fluids (e.g., oil) and longer pipelines even with relatively low viscosity fluid such as water. But even on lower frictional pressure drop systems the line pack phenomenon can be observed.

Pipeline hydraulic engineers familiar with water hammer typically have a strong appreciation for line pack. On the other hand, plant system engineers who deal with smaller scale piping systems often do not have the same appreciation.

Interestingly, few cases for field measurements of line pack are found in the literature. Numerical predictions are easier to find (as referenced in the section introduction above). Even with limited field measurements, line packing is common knowledge among fluid transient engineers.

Along with Liou's explanation [13], quoted above, he goes deeper into the basic mechanism that causes line pack. Specifically, when a valve is instantly closed, and the first water hammer wave propagates inside the pipe, it fails to bring the fluid to a zero velocity throughout the entire pipe except right at the closed valve. The fluid behind the wave thus still has forward velocity towards the valve and that causes the pressure at the valve to slowly increase above the Joukowsky pressure (Eq. 1). This pressure increase behind the wave is the phenomenon known as line pack.

### Example 3: Line Pack

To better understand line pack, consider the horizontal pipe shown in Fig. 3, using the characteristics listed in Table 1. This example expands on Example 1. Where the simplified Example 1 above neglected friction effects, this example considers the Darcy (Moody) friction factor, *f*, with respect to pressure changes. Fig. 4 shows the simulation results for 500 seconds. The piping was modeled using 100 sections, and the valve was modeled as a fixed flow rate which drops to zero instantly.

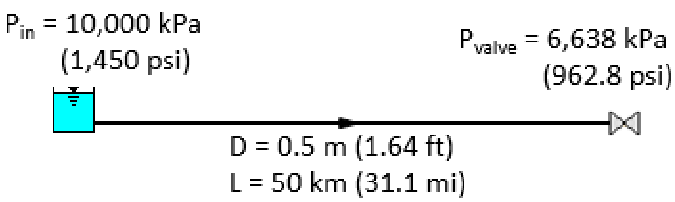


**Figure 3: Example 3 – Horizontal pipe system description**

**Table 1: Input data for Example 3, assuming instantaneous valve closure**

| | |
|---|---|
| $L$ | 50 km (31.1 miles) |
| $D$ | 0.5 m (1.64 ft) |
| $a$ | 1,291 m/s (4,236 ft/s) |
| $f$ | 0.018 |
| $Q$ | 0.4 m$^3$/s (14.1 ft$^3$/s), 1,440 m$^3$/hr (6,340 gpm) |
| $\Delta V$ | -2.04 m/s (-6.68 ft/s) |
| $P_{in}$ | 10,000 kPa (1,450 psi), fixed |
| $P_{valve}$ | 6,638 kPa (962.8 psi), upstream initial pressure |
| $\Delta P_{pipe}$ | 3,362 kPa (487.6 psi), initial pipe pressure drop |
| $\rho$ | 900 kg/m$^3$ (56.2 lbm/ft$^3$) |

Consider what the line pressure will be once the valve has closed and all transients have steadied out. The answer is trivial: the line pressure will be 10,000 kPa (1,450 psi) at all points based on the upstream pressure and no flow or elevation change. How much did the final pressure increase at the valve? The pressure increased 3,362 kPa (6,638 to 10,000) or 487 psi (962.8 to 1,450). This pressure increase does not depend on water hammer and is a result of the recovery of pressure at the valve previously lost to friction when the pipeline was flowing. We will therefore call this the "friction recovery pressure", $\Delta P_{fr}$.

Eq. 3 predicts the maximum Joukowsky pressure increase at the valve due to water hammer ($\Delta P_J$) is 2,367 kPa (343 psi). See Example 1. And that is where many engineers stop when evaluating water hammer. But that neglects friction recovery pressure. To get a maximum possible pressure increase, one needs to account for the recovery of pressure from friction. One quick estimate is to add the friction recovery pressure to the Eq. 3 pressure, since both contribute to the pressure increase in the piping. Adding the two together obtains a maximum possible pressure increase of:

$$\Delta P_{max} = \Delta P_J + \Delta P_{fr} = 5{,}729 \text{ kPa (831 psi)} \tag{8}$$

This estimate provides a very conservative answer, where the pressures to be added occur at two different times: one time occurs when the valve closes; the other time occurs later when the system comes to equilibrium.

Note two conclusions from this analysis. First, the friction recovery pressure is different at every point along the pipeline and obtains a maximum at the valve since the valve is the furthest point along the pipeline and thus experiences the most friction pressure drop. Second, the friction recovery pressure is a quick and conservative estimate of the maximum pressure. In reality, by the time the friction pressure recovery occurs, the Eq. 3 pressure spike has attenuated (see Leslie and Tijsseling [15] for more on friction and attenuation). The new method discussed by Liou [13] endeavors to predict the sum of the friction recovery pressure and the attenuated Eq. 3 spike. It can be shown that Liou's method predicts the following:

$$\Delta P_{max} = 5{,}311 \text{ kPa (770 psi)} \tag{9}$$

This value is lower than the quick and conservative method of Eq. 8. How does it compare to an actual numerical simulation? Figs. 4 and 5 show the answer to that. The Liou method predicts the peak pressure exceedingly well.

Fig. 4 shows numerical analysis results for the first 500 seconds, and also shows the predicted pressure increase from the Joukowsky equation (Eq. 3). Fig. 5 shows these same numerical analysis results for the first 200 seconds. Additionally, shown in Fig. 5 is the sudden pressure increase expressed by the Joukowsky equation (Eq. 3), the pressure increases from line pack, the Liou [13] method for calculating pressure increases, and the friction recovery pressure increase. Note that the sum of the Eq. 3 pressure from the Joukowsky equation and the friction recovery pressure is conservatively higher than the numerical simulation or Liou's pressure prediction. Finally, it is clear from Figs. 4 and 5 that the actual pressure increase exceeds that predicted from Eq. 3.

### Further Comments on Line Pack

As seen in Fig. 5, the maximum pressure using the friction recovery pressure (12,367 kPa, 1,794 psi) is conservatively higher than the actual maximum (11,967 kPa, 1,763 psi) and the Liou method (11,949 kPa, 1,733 psi). Note that this system is based on oil, a relatively high viscosity fluid. For practical applications, use of the friction recovery pressure results in conservative maximum pressures.

As mentioned earlier, published experimental/field test evidence for line pack is difficult to find. One of the signs of line pack is the increasing pressure right after the valve has instantly closed. This phenomenon can be observed in Figs. 4 and 5 after valve closure until the pressure peak at about 75 seconds. Interestingly, Fig. 1 is redrawn as Fig. 6 using a different scale for emphasis where line pack is clearly identified. Note that the friction recovery pressure is about 5 m (16 ft) of water in Fig. 6. Martin's data uses water and a relatively short pipe (102 m) showing that line pack also happens in short, low viscosity systems.

For completeness, the Fig. 1 simulation results were compared with and without the varying tank pressure reported by Bergant et al. [1, Fig. 4]. The line pack in the simulated results was very similar between the two cases. This further confirms that the 5 m (16 ft) pressure rise above Eq. 1 in Fig. 6 is a result of line pack.

### Practical Line Pack Advice for Engineers

We have shown that Eq. 3 does not predict maximum pressures when line pack occurs. For any system with appreciable frictional pressure drop, the line pack effect will be pronounced. A quick and conservative way to estimate maximum line pressures is to add the Eq. 3 Joukowsky pressure increase to the friction recovery pressure.

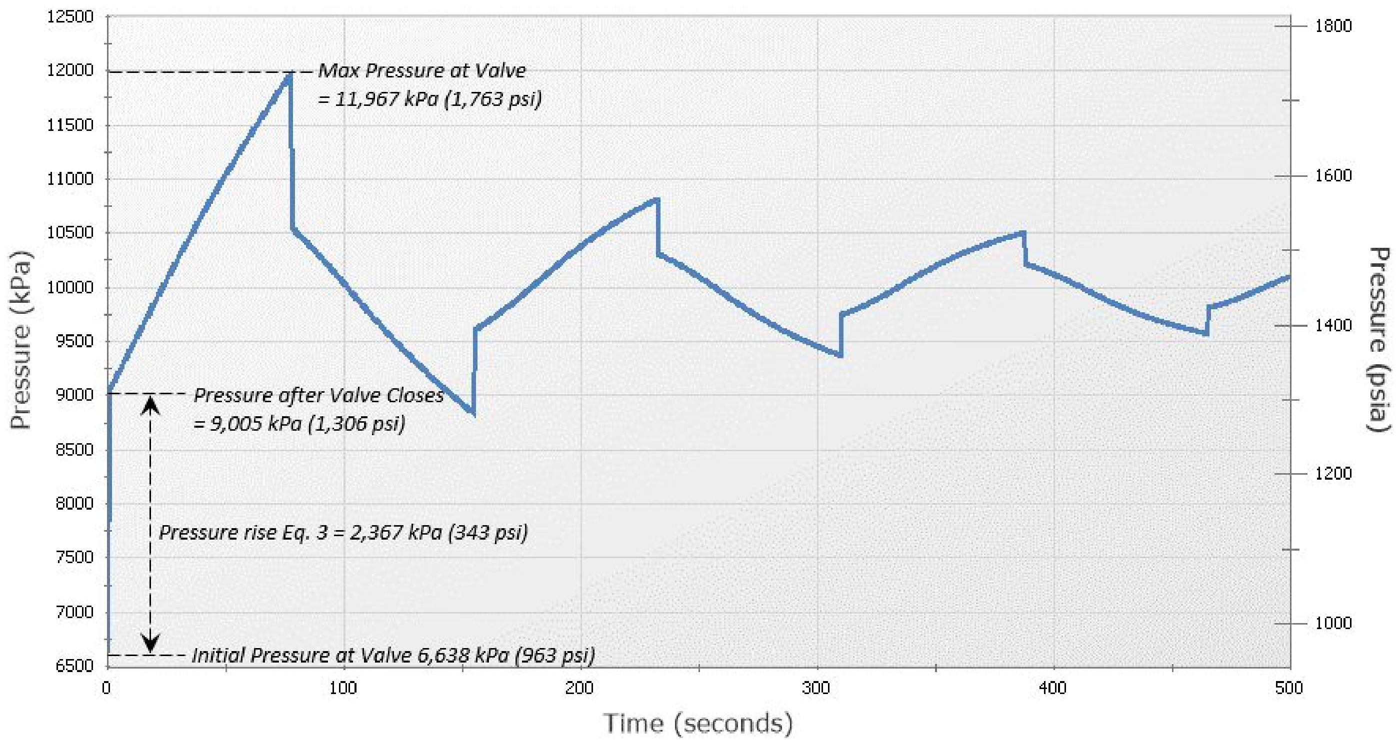


**Figure 4: Example 3 - Predicted pressure transient at the valve for the system shown in Fig. 3 for 500 s**

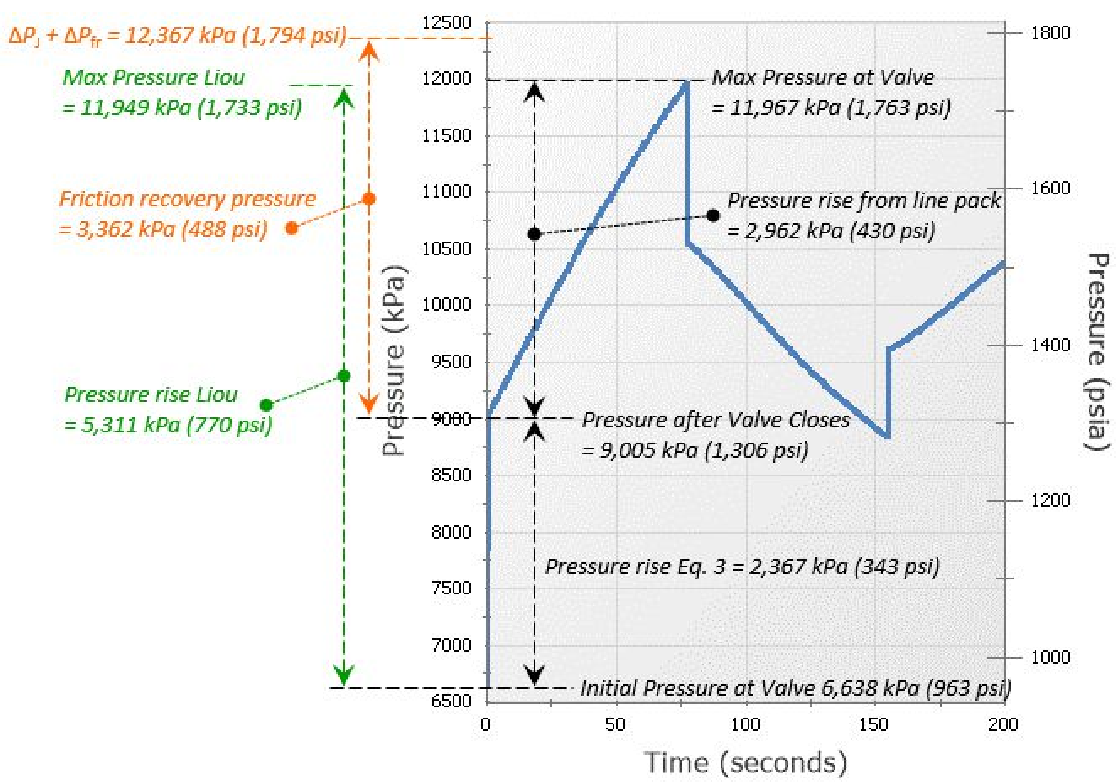


**Figure 5: Example 3 – Predicted pressure transient at the valve from the system shown in Fig. 3 for 200 s with additional details on the various pressure rise estimation methods**

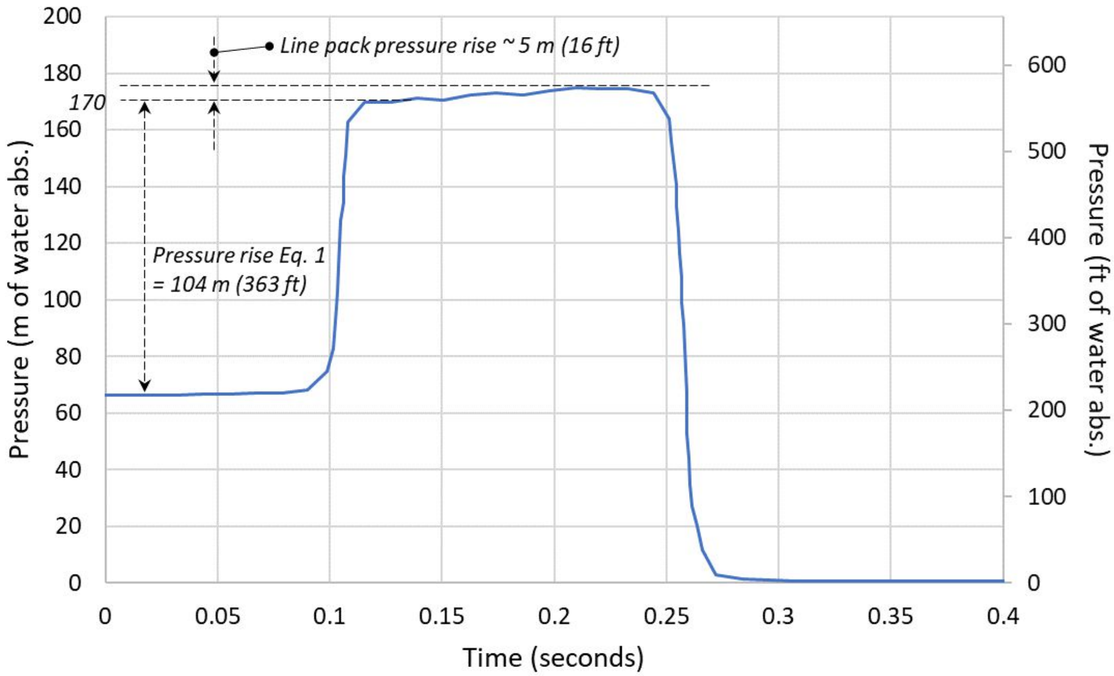


**Figure 6: Example 2 - Evidence of line pack effect in experimental results (Martin [12])**

## PIPING SYSTEM PRESSURE WAVE REFLECTIONS

Water hammer wave reflections can occur for many reasons including:

- Branching / tees
- Piping diameter changes
- Valves and fittings which result in any diameter change and/or introduce a local pressure drop
- Dead ends
- Pumps
- Tanks or reservoirs
- Accumulators
- Blockages in pipes
- Leaks in pipes
- Vibrating elbows
- Entrapped air pockets
- Wave speed changes due to piping material or wall thickness changes
- Frictional characteristics changes

In all of these cases, an abrupt change in the wave propagation occurs at a transition. In any of these cases where a transition of material or structural characteristics occurs, both a reflected wave and a transmitted wave will also occur at that transition (Leishear [6]).

All following examples use Applied Flow Technology [11] and are based on water.

### Example 4: Reflections in Piping Networks

In the past, some believed that piping networks always reduced the maximum pressure of water hammer waves (Karney and McInnis [16]). While this may be true in some cases, Karney and McInnis call this belief "transient folklore". They considered this comparative example for two piping systems, as described in Figs. 7 and 8. An instant valve closure in the straight pipe system of Fig. 7 yields an initial pressure increase as predicted by Eq. 3. What about the networked system in Fig. 8 with instant valve closure?

Fig. 9 shows simulation results which, as expected, yield essentially the same results as Karney and McInnis [16]. Note how network system pressures exceed straight system pressures in Fig. 9, where results are presented for straight and networked piping systems for instant valve closures. Clearly, the networked system in Fig. 8 yields higher pressures than the straight system of Fig. 7. These results show that, in some cases, networked systems yield pressure increases significantly greater than the maximum Joukowsky prediction (Eq. 3).

Eq. 3 obtains a pressure increase of 130 m (427 ft) of water for this system. This result is also shown in Fig. 9. The peak pressure of 191 m (627 ft) is also shown. This pressure increase is 47% higher than the increase predicted from the Joukowsky equation.

Note, to achieve the results shown in Fig. 9, a valve *Cv* of 1,390 was used in the system from Fig. 7, and a *Cv* of 1,384 was used in the system from Fig. 8. The valve *Cv's* need to be different to match the head loss values in Figs. 7 and 8 as well as to maintain the specified flow rates. The system was modeled with 25 sections in the shortest pipe.

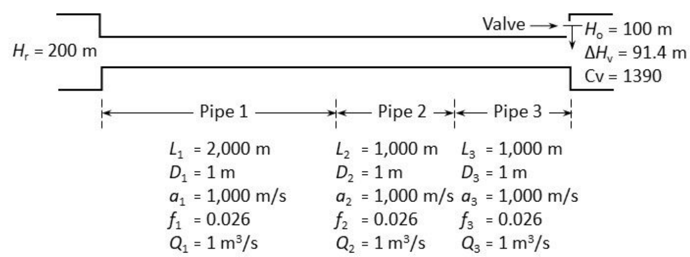


**Figure 7: Example 4 – Straight piping system**

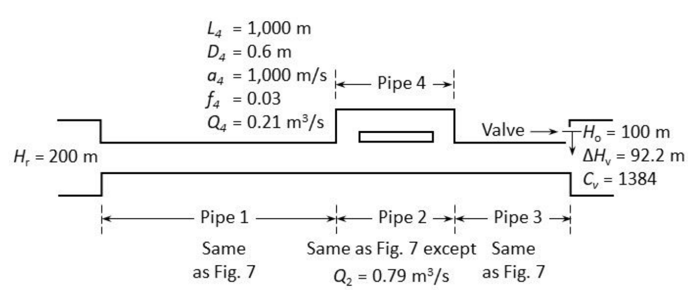


**Figure 8: Example 4 - Networked piping system**

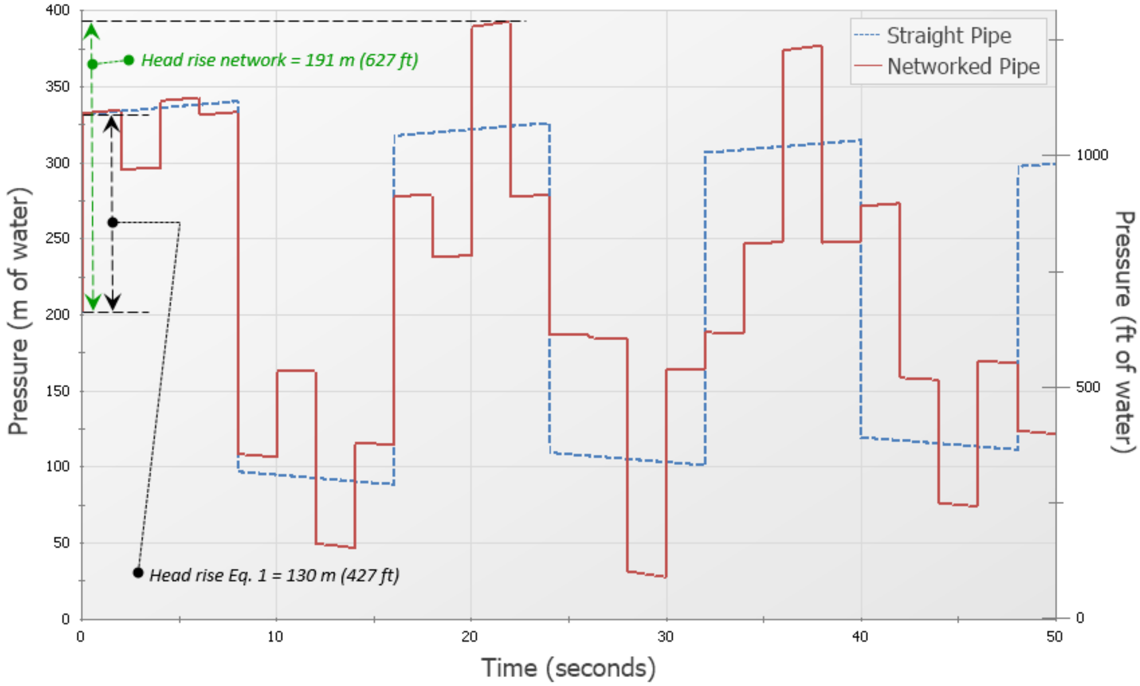


**Figure 9: Example 4 - Simulation results using [11] for Figs. 7 and 8 at the valve for straight and networked systems (adapted from Karney and McInnis [16])**

### Example 5: Reflections From Diameter Changes

This example is the same as Fig. 7 except that the diameter of Pipe 2 has been increased to 1.17 m and the valve *Cv* has been changed to 1,381 to match the same overall flowrate as Fig. 7. The valve is again closed instantly. Note that the diameter of 1.17 m was chosen to give the same effective flow area as the sum of the areas from Pipes 2 and 4 from Fig. 8. See system in Fig. 10.

Fig. 12 shows the results. Similar to Example 4, Eq. 3 is exceeded. Also, the close similarity of results between Figs. 9 and 12 leads one to ask whether the pressures in Example 4 are more a result of the pipe network as discussed in [16] or the effective area change at the branch closest to the valve (Fig. 8). Having the same wave speed in Fig. 8 Pipes 2 and 4 contribute to this result. Different wave speeds in these pipes would yield a more complicated transient than shown in Fig. 9. This system is modeled with 25 sections in the shortest pipe.

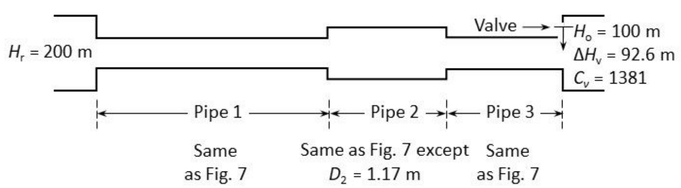


**Figure 10: Example 5 - Piping system with diameter change**

### Example 6: Reflections From Branch With Dead End

This example is the same as Fig. 7 except that there is a branch with a dead end (see Fig. 11). The valve is again closed instantly.

Fig. 13 shows results. Similar to previous examples, Eq. 3 is exceeded. This system is modeled with 1 section in Pipe 4 (Fig. 11) and a minimum of 25 sections in all other pipes.

### Example 7: Reflections From Gas Accumulator

This example is the same as Fig. 7 except that there is an inline accumulator 40 meters from the valve. The initial gas volume was 20,000 liters (5,280 gal) and a polytropic constant of 1.2. See Fig. 14. The valve is again closed instantly. This system is modeled with 1 section in Pipe 3 (Fig. 14) and a minimum of 25 sections in all other pipes.

Fig. 16 shows the results. Similar to previous examples, the predicted maximum pressure from Eq. 3 is exceeded.

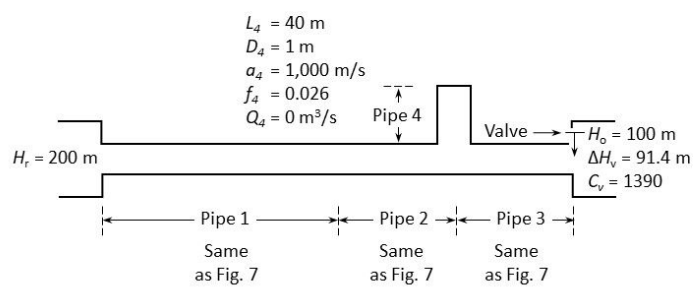


**Figure 11: Example 6 - Piping system with a branch and dead end**

### Further Comments on Systems With Wave Reflections

Real systems often have many pressure wave reflection points which lead to complicated wave propagation patterns. Fig. 15 shows simulation results at various times for the Fig. 10 system, with wave speed in Pipe 2 changed from 1,000 to 900 m/s. It is clear that the pressure and flow distribution becomes increasingly more complicated as time progresses even though the initial transient began as a single wave due to instant valve closure. This system is modeled with a minimum of 500 sections in each pipe in order to show a steep wave front in Fig. 15. The wave speed of 900 m/s in Pipe 2 was chosen to introduce asymmetry in the wave reflection times in Pipe 1 compared to Pipes 2 and 3 when combined.

### Practical Pipe Reflection Advice for Engineers

Conclusively, Eq. 3 does not predict maximum pressures when certain pipe reflections occur. It is much harder to estimate the magnitude of pressure surge at a reflection as there are various types of reflections, as shown in Examples 4-7. Consult Parmakian [17] for analytical approximations for water hammer at reflections. Strongly consider using numerical methods.

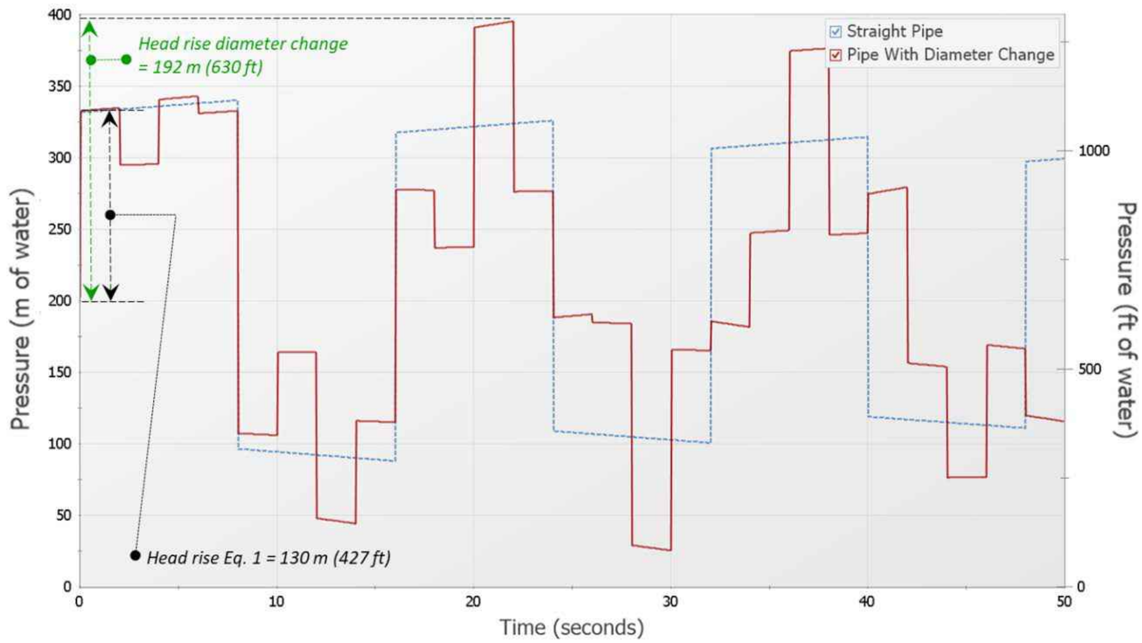


**Figure 12: Example 5 - Simulation results for Figs. 7 and 10 at the valve for straight pipe and pipe with diameter change systems.**

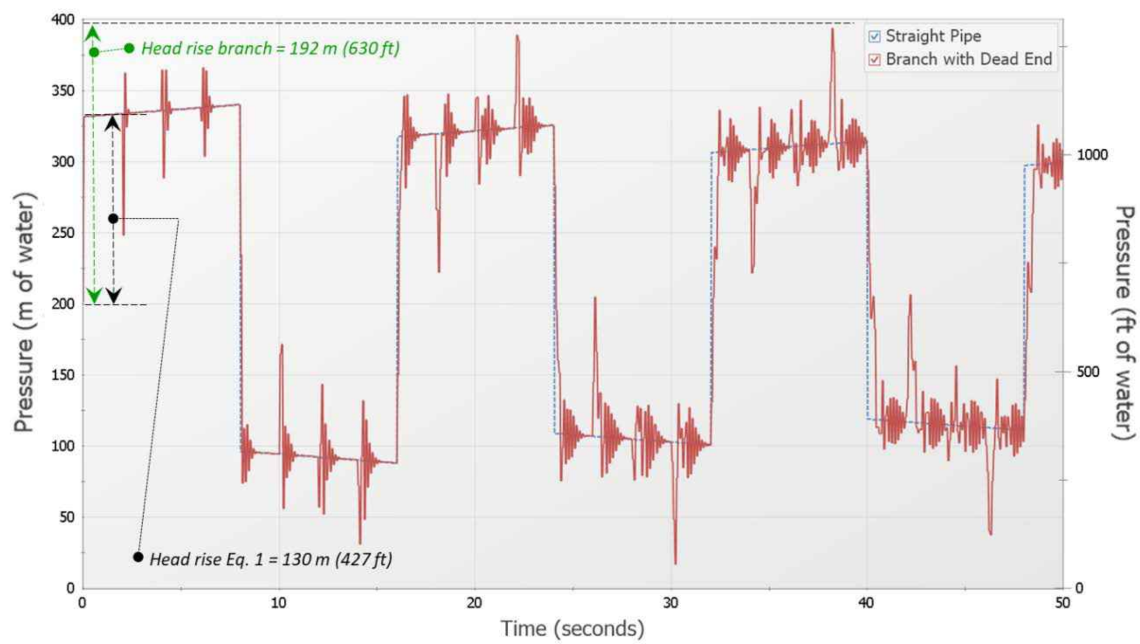


**Figure 13: Example 6 - Simulation results for Figs. 7 and 11 at the valve for straight pipe and branch with dead end systems.**

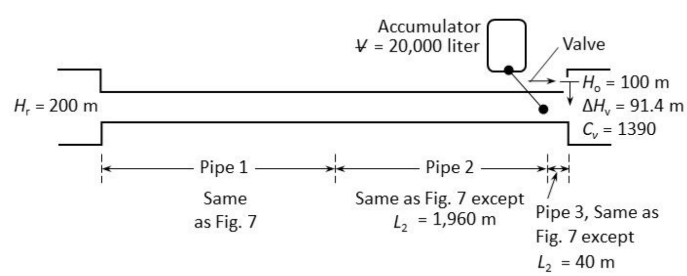


**Figure 14: Example 7 - Piping system with a gas accumulator**

### Dynamic Stresses

Although outside the scope of this work, dynamic stresses merit further comment. When pressures are suddenly applied due to water hammer, the expected static stress is multiplied by a dynamic load factor (DLF) to obtain the actual dynamic stress exerted on the piping. For elastic hoop stresses, DLF < 4 when a steep fronted water hammer wave travels along the bore of a pipe (Leishear [6]). For elastic bending stresses, DLF < 2 for a single elbow, but the DLF can be increased up to DLF < 4 for tight U-bend axial stresses and Z-bend bending stresses. As the pressure is more gradually applied, the DLF approaches 1 for hoop stresses and bending stresses, which is the case for line pack.

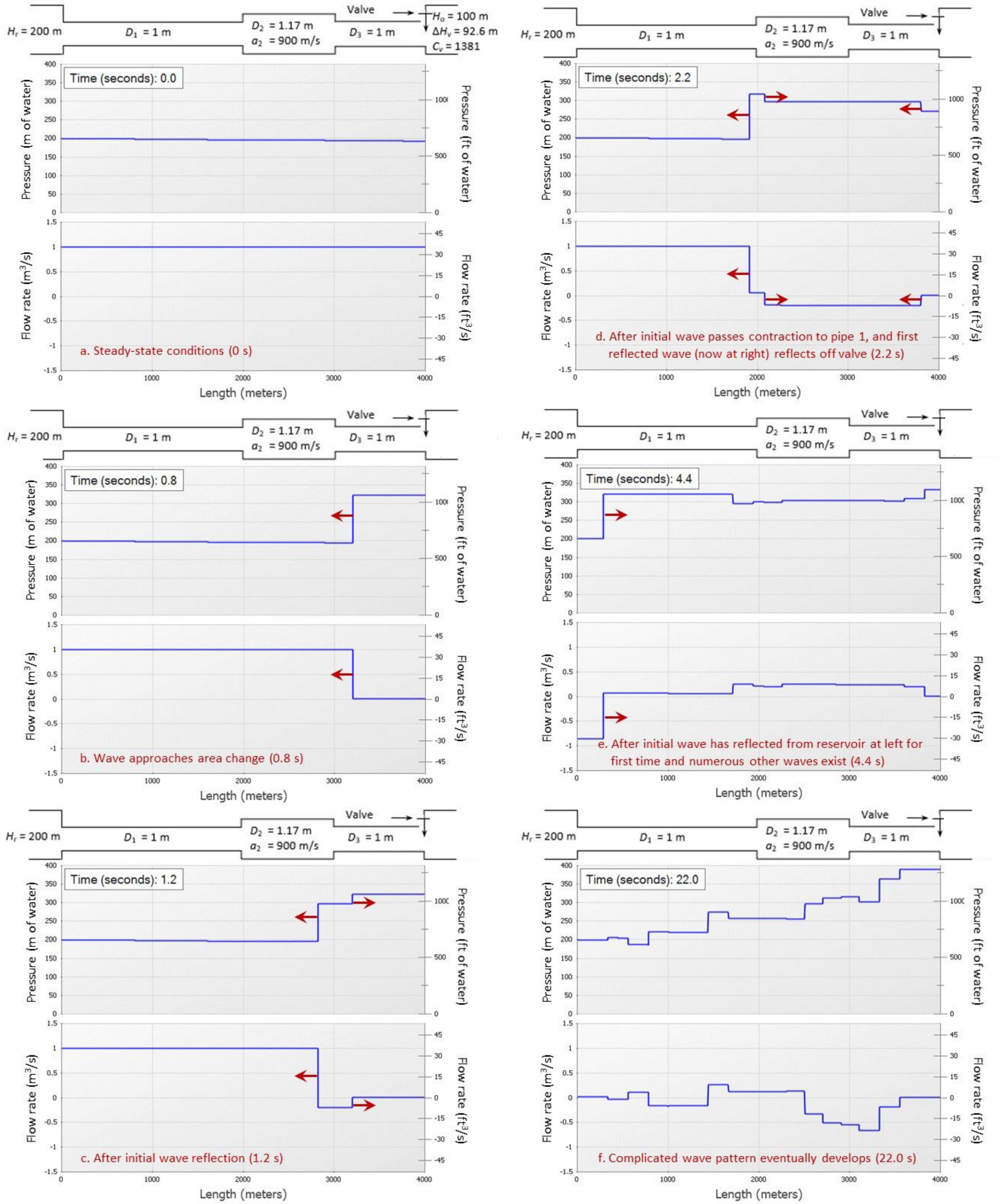


**Figure 15: Simulation results at various simulation times for the Fig. 10 system (with wave speed changed in Pipe 2 to 900 m/s) showing pressure and flow rate profiles and how wave patterns become more complicated over time**

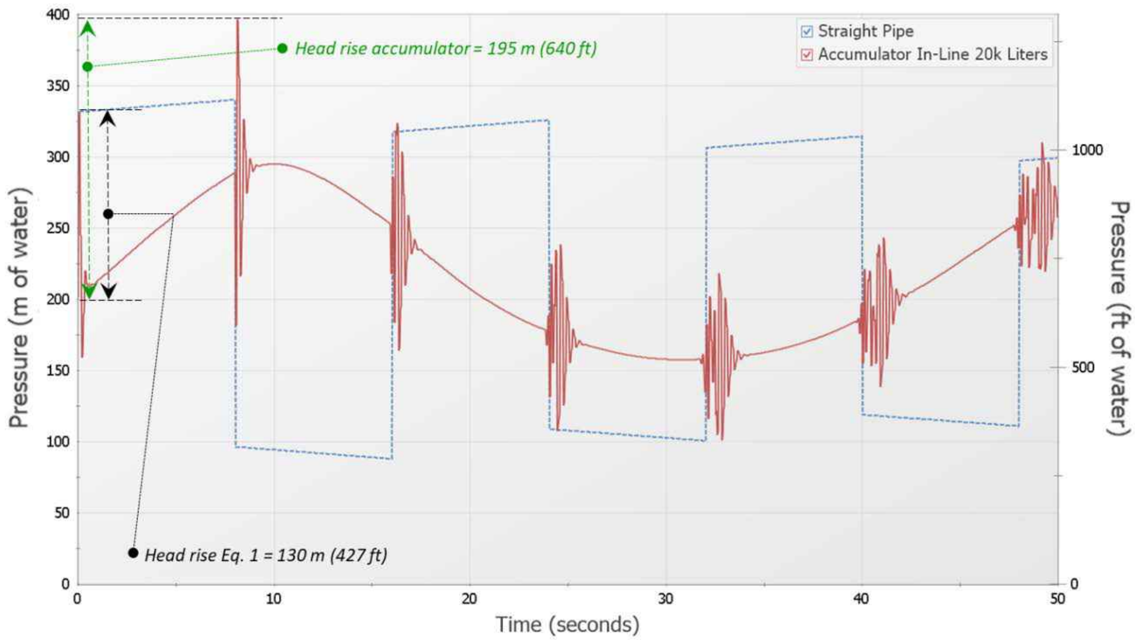


**Figure 16: Example 7 - Simulation results for Figs. 7 and 14 at the valve for straight pipe and gas accumulator systems**

## CONCLUSIONS

The Joukowsky equation should be used judiciously in piping systems for several conditions:

1. Piping systems that contain tees
2. Piping systems that contain changes in pipe diameter, pipe material, pipe wall thickness, or frictional coefficients
3. Piping systems where increased pressures due to line pack may be an issue (examples are long pipelines and/or higher viscosity fluids)
4. Systems where pressures drop to the vapor pressure of the liquid in the piping system

All in all, when these complex conditions are present in piping systems, numerical methods are preferred to the simplified Joukowsky equation to prevent a misunderstanding of system performance. Significant mistakes can be made by using the simplified Joukowsky equation without a more complete awareness of its limitations.

## ACKNOWLEDGEMENTS

The authors would like to acknowledge the assistance of Dylan Witte of Purple Mountain Technology Group for researching Example 2.

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